Let Ae^{kt} be the decay eq'n for methane

Then let

$$W_M(t) = A\int_0^t e^{kx}\,dx$$

~~$W_M(t)$ gives the total CO_2 equivelence of methane over a time t, if~~

Let $W_C(t)$ be the CO_2 equivalent of CO_2 over the time t, with $W_C(1)=1$.

Then $W_C(t) = t\cdot 1 = t$, and for methane, the Global Warming potential $GWP(t)$ is given by

$$W_M(t) = GWP(t)\cdot W_C(t)$$

$$GWP(t) = \frac{1}{t}\,W_M(t)$$

Given $GWP(20) = 81$, $GWP(100) = 28$

① $\frac{1}{20} A \int_0^{20} e^{kx}\, dx = 81$

② $\frac{1}{100} A \int_0^{100} e^{kx}\, dx = 28$

① $\frac{1}{20} \frac{A}{k} \left(e^{kx} \Big]_0^{20} \right) = 81$

$\frac{1}{20} \frac{A}{k} (e^{20k} - 1) = 81$

$$\boxed{\frac{A}{k} (e^{20k} - 1) = 1620}$$ ③

② $\frac{1}{100} \frac{A}{k} \left(e^{kx} \Big]_0^{100} \right) = 28$

$\frac{1}{100} \frac{A}{k} (e^{100k} - 1) = 28$

$$\boxed{\frac{A}{k} (e^{100k} - 1) = 2800}$$

$$\frac{2800}{1620}\,\cancel{\frac{A}{k}}\left(e^{20k}-1\right)=\cancel{\frac{A}{k}}\left(e^{100k}-1\right)$$

$$(1.728395)(e^{20k}-1)=e^{100k}-1$$

Let $u=e^{20k}$

$$(1.728395)(u-1)=u^5-1$$

$$u^5-1.728395u+0.728395=0$$

Using Wolfram Alpha:

$$u=0.42993$$

$$e^{20k}=0.42993$$

$$k=\frac{1}{20}(\ln(0.42993))=-0.0422066$$

(5) $\boxed{k=-0.0422066}$

Plug (5) into (3) and solve for A

~~A = 2800 x k~~ $\qquad A=\frac{1620\times k}{(e^{20k}-1)}$

⑥ $A = 119.94095 \approx 120$

Now, we want $GWP(0)$ ~

$$GWP(0) = \lim_{t \to 0} GWP(t)$$

$$= \lim_{t \to 0} \frac{1}{t} w_M(t)$$

$$= \lim_{t \to 0} \frac{1}{t} A \int_0^t e^{kx}\, dx$$

$$= \lim_{t \to 0} \frac{1}{t} \frac{A}{k} (e^{kt} - 1)$$

$$= \lim_{t \to 0} \frac{A(e^{kt} - 1)}{kt} \qquad \frac{0}{0}$$

use L'hopital's rule

$$= \lim_{t \to 0} \frac{\cancel{k} A e^{kt}}{\cancel{k}}$$

$$= \lim_{t \to 0} A e^{kt} = A \times 1 = A$$

$$\boxed{GWP(0) = A = 120}$$